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Smarandache 对偶函数的一个计算公式

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摘要　对于任意正整数 n,著名的 Smarandache 对偶函数 $s^*(n)$定义为使得 $m!/n$ 最大的正整数 m,利用初等方法研究了关于对偶函数$\sum_{d/n}s^*(d)$,并给出了一个计算公式。

关键词　Smarandache 函数;对偶函数;计算公式
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A Study of Smarandache Dual Function Calculation Formula

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Abstract:For any postive integer n, the famous Smarandache function is defined as making $m!/n$ the largest positive integer m. In the paper, the dual function $\sum_{d/n}s^*(d)$ is studied and a calculation formula is given by using an elementary method.

Key words:smarandache function; dual function; formula

Smarandache 函数是由罗马尼亚著名数论专家 J.Sandor 在文献[1]中首次提出的,并研究了它的各种初等性质,获得了一系列重要结论。关于这个问题,不少学者也做过研究,并且得到了一些有意义的结论。文献[2]中,李洁研究了一个包含 $s^*(n)$的无穷级数的敛散性,并获得了一个恒等式。即就是对任意的实数 $\alpha\leqslant1$,无穷级数 $\sum_{n=1}^{\infty}\frac{s^*(n)}{n^\alpha}$是发散的,当 $\alpha>1$ 时,是收敛的,而且:$\sum_{n=1}^{\infty}\frac{s^*(n)}{n^\alpha}=\zeta(\alpha)\cdot\sum_{n=1}^{\infty}\frac{1}{(n!)^\alpha}$,式中 $\zeta(\alpha)$是 Riemann-zeta 函数。注意到 $\zeta(2)=\frac{\pi}{6}$,$\lim_{s\to1}(s-1)\zeta(s)=1$,及$\sum_{n=1}^{\infty}\frac{1}{n!}=e-1$,由上式可以推出:$\sum_{n=1}^{\infty}\frac{s^*(n)}{n^2}=\frac{\pi^2}{6}\sum_{n=1}^{\infty}\frac{1}{(n!)^2}$,此外,文献[3]中,还利用初等方法获得了较强的渐近公式:$\sum_{n\leqslant1}s^*(n)=(e-1)x+o(\frac{\mathrm{In}^2(x)}{(\mathrm{InIn}x)^2})$。;文献[4]给出了一个包含 Smarandache 对偶函数的方程所有正整数解,文献[5]给出了一个包含 Smarandache 函数的对偶方程的正整数解。关于这一函数以及有关内容也可以参阅文献[6～8]。

本文利用初等方法研究 Smarandache 对偶函数$\sum_{d/n}s^*(d)$,并给出了一个计算公式。

1　定理及结论

定理　对于正整数 n,关于 Smarandache 对偶函数$\sum_{d/n}s^*(d)$的计算公式为:

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$$\sum_{d/n} s^*(d)=\begin{cases}(\alpha_1+1)(\alpha_2+1)\cdots(\alpha_k+1), & n=p_1^{\alpha_1}p_2^{\alpha_2}\cdots p_k^{\alpha_k},p_i \text{ 是奇素数}\\ (2\alpha+1)(\alpha_1+1)(\alpha_2+1)\cdots(\alpha_k+1), & n=2^{\alpha}p_1^{\alpha_1}p_2^{\alpha_2}\cdots p_k^{\alpha_k},p_1\neq 3\\ (2\alpha+1+\alpha_1+3\alpha\alpha_1)(\alpha_2+1)\cdots(\alpha_k+1), & n=2^{\alpha}p_1^{\alpha_1}p_2^{\alpha_2}\cdots p_k^{\alpha_k},p_1=3,\alpha=1,2\end{cases}$$

式中：p_i 为互不相同的奇素数；α_i，α 是正整数。

2 定理的证明

对于任意正整数 n，当 n 为奇数时，此时对任意 d/n，显然 2！不整除 n，所以 $s^*(d)=1$，则 $\sum_{d/n}s^*(d)=\sum_{d/n}1=d(n)$，$d(n)$ 表示 Dirichlet 除数函数。因此，分以下几种情况来讨论，为了方便，令 $n=p_1^{\alpha_1}p_2^{\alpha_2}\cdots p_k^{\alpha_k}$ 为 n 的标准素因子分解式，式中 p_i 为奇素数。

2.1 $n=p_1^{\alpha_1}p_2^{\alpha_2}\cdots p_k^{\alpha_k}$

$$\sum_{d/n}s^*(d)=\sum_{d/p_1^{\alpha_1}p_2^{\alpha_2}\cdots p_k^{\alpha_k}}1=d(p_1^{\alpha_1}p_2^{\alpha_2}\cdots p_k^{\alpha_k})=(\alpha_1+1)(\alpha_2+1)\cdots(\alpha_k+1)。$$

2.2 $n=2p_1^{\alpha_1}p_2^{\alpha_2}\cdots p_k^{\alpha_k}$

1）当 $p_1\neq 3$ 时，则有：

$$\sum_{d/n}s^*(d)=\sum_{d/p_1^{\alpha_1}p_2^{\alpha_2}\cdots p_k^{\alpha_k}}s^*(d)+\sum_{d/p_1^{\alpha_1}p_2^{\alpha_2}\cdots p_k^{\alpha_k}}s^*(2d)=\sum_{d/p_1^{\alpha_1}p_2^{\alpha_2}\cdots p_k^{\alpha_k}}1+\sum_{d/p_1^{\alpha_1}p_2^{\alpha_2}\cdots p_k^{\alpha_k}}2=3d(p_1^{\alpha_1}p_2^{\alpha_2}\cdots p_k^{\alpha_k})=3(\alpha_1+1)(\alpha_2+1)\cdots(\alpha_k+1)。$$

2）当 $p=3$ 时，则有：

$$\sum_{d/n}s^*(d)=\sum_{d/3^{\alpha_1}p_2^{\alpha_2}\cdots p_k^{\alpha_k}}s^*(d)+\sum_{d/3^{\alpha_1}p_2^{\alpha_2}\cdots p_k^{\alpha_k}}s^*(2d)=(\alpha_1+1)(\alpha_2+1)\cdots(\alpha_k+1)+\sum_{d/p_2^{\alpha_2}\cdots p_k^{\alpha_k}}s^*(2d)+\sum_{d/p_2^{\alpha_2}\cdots p_k^{\alpha_k}}s^*(2\times 3d)+\cdots+\sum_{d/p_2^{\alpha_2}\cdots p_k^{\alpha_k}}s^*(2\times 3^{\alpha_1}d)=(\alpha_1+1)(\alpha_2+1)\cdots(\alpha_k+1)+2(\alpha_2+1)\cdots(\alpha_k+1)+3\alpha_1(\alpha_2+1)\cdots(\alpha_k+1)=(4\alpha_1+3)(\alpha_2+1)\cdots(\alpha_k+1)。$$

2.3 $n=2^2p_1^{\alpha_1}p_2^{\alpha_2}\cdots p_k^{\alpha_k}$

1）当 $p_1\neq 3$ 时，则有：

$$\sum_{d/n}s^*(d)=\sum_{d/p_1^{\alpha_1}p_2^{\alpha_2}\cdots p_k^{\alpha_k}}s^*(d)+\sum_{d/p_1^{\alpha_1}p_2^{\alpha_2}\cdots p_k^{\alpha_k}}s^*(2d)+\sum_{d/p_1^{\alpha_1}p_2^{\alpha_2}\cdots p_k^{\alpha_k}}s^*(4d)=(\alpha_1+1)(\alpha_2+1)\cdots(\alpha_k+1)+2(\alpha_1+1)(\alpha_2+1)\cdots(\alpha_k+1)+2(\alpha_1+1)(\alpha_2+1)\cdots(\alpha_k+1)=5(\alpha_1+1)(\alpha_2+1)\cdots(\alpha_k+1)。$$

2）当 $p_1=3$ 时，则有：

$$\sum_{d/n}s^*(d)=\sum_{d/3^{\alpha_1}p_2^{\alpha_2}\cdots p_k^{\alpha_k}}s^*(d)+\sum_{d/3^{\alpha_1}p_2^{\alpha_2}\cdots p_k^{\alpha_k}}s^*(2d)+\sum_{d/3^{\alpha_1}p_2^{\alpha_2}\cdots p_k^{\alpha_k}}s^*(4d)=(\alpha_1+1)(\alpha_2+1)\cdots(\alpha_k+1)+\sum_{d/p_2^{\alpha_2}\cdots p_k^{\alpha_k}}s^*(2d)+\sum_{d/p_2^{\alpha_2}\cdots p_k^{\alpha_k}}s^*(2\times 3d)+\cdots+\sum_{d/p_2^{\alpha_2}\cdots p_k^{\alpha_k}}s^*(2\times 3^{\alpha_1}d)+\sum_{d/p_2^{\alpha_2}\cdots p_k^{\alpha_k}}s^*(4d)+\cdots+\sum_{d/p_2^{\alpha_2}\cdots p_k^{\alpha_k}}s^*(4\times 3^{\alpha_1}d)=(\alpha_1+1)(\alpha_2+1)\cdots(\alpha_k+1)+2(\alpha_2+1)\cdots(\alpha_k+1)+3\alpha_1(\alpha_2+1)\cdots(\alpha_k+1)+2(\alpha_2+1)\cdots(\alpha_k+1)+3\alpha_1(\alpha_2+1)\cdots(\alpha_k+1)=(7\alpha_1+5)(\alpha_2+1)\cdots(\alpha_k+1)。$$

2.4 $n=2^{\alpha}p_1^{\alpha_1}p_2^{\alpha_2}\cdots p_k^{\alpha_k}$

1）当 $p_1\neq 3$ 时，则有：

$$\sum_{d/n}s^*(d)=\sum_{d/p_1^{\alpha_1}p_2^{\alpha_2}\cdots p_k^{\alpha_k}}s^*(d)+\sum_{d/p_1^{\alpha_1}p_2^{\alpha_2}\cdots p_k^{\alpha_k}}s^*(2d)+\cdots+\sum_{d/p_1^{\alpha_1}p_2^{\alpha_2}\cdots p_k^{\alpha_k}}s^*(2^{\alpha}d)=(\alpha_1+1)(\alpha_2+1)\cdots(\alpha_k+1)+2\alpha(\alpha_1+1)(\alpha_2+1)\cdots(\alpha_k+1)=(2\alpha+1)(\alpha_1+1)(\alpha_2+1)\cdots(\alpha_k+1)。$$

2）当 $p_1=3$，$\alpha=1,2$ 时，则有：

$$\sum_{d/n}s^*(d)=\sum_{d/3^{\alpha_1}p_2^{\alpha_2}\cdots p_k^{\alpha_k}}s^*(d)+\sum_{d/3^{\alpha_1}p_2^{\alpha_2}\cdots p_k^{\alpha_k}}s^*(2d)+\cdots+\sum_{d/3^{\alpha_1}p_2^{\alpha_2}\cdots p_k^{\alpha_k}}s^*(2^{\alpha}d)=(\alpha_1+1)(\alpha_2+1)\cdots(\alpha_k+1)+\sum_{d/p_2^{\alpha_2}\cdots p_k^{\alpha_k}}s^*(2d)+\sum_{d/p_2^{\alpha_2}\cdots p_k^{\alpha_k}}s^*(2\times 3d)+\cdots+\sum_{d/p_2^{\alpha_2}\cdots p_k^{\alpha_k}}s^*(2\times 3^{\alpha_1}d)+\cdots+\sum_{d/p_2^{\alpha_2}\cdots p_k^{\alpha_k}}s^*(2^{\alpha}d)+\cdots+$$

$$\sum_{d/p_2^{\alpha_2}\cdots p_k^{\alpha_k}} s^*(2^{\alpha}\times 3^{\alpha_1} d) = (\alpha_1+1)(\alpha_2+1)\cdots(\alpha_k+1)+2\alpha(\alpha_2+1)\cdots(\alpha_k+1)+3\alpha\alpha_1(\alpha_2+1)\cdots(\alpha_k+1)=(2\alpha+1+\alpha_1+3\alpha\alpha_1)(\alpha_2+1)\cdots(\alpha_k+1)。$$

这样就完成了定理的证明。

3 结语

在 1991 年美国研究出版社出版的《只有问题,没有解答》一书中,F.Smarandache 教授提出了 105 个关于特殊数列、算术函数等未解决的数学问题及猜想,而 Smarandache 对偶函数就是其中一类函数,关于此函数,不少学者专家对此进行了深入研究,并得到了具有理论价值的研究成果。本文通过对对偶函数的研究,得到了此计算公式,更方便进一步研究包含对偶函数的其它问题,并探讨其他 Smarandache 函数的对偶函数的一些性质与结论。

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